Universidad Rey Juan Carlos

Topic 3: Signal Representation in Communication Systems

Academic Year 2013 - 2014

P1.- How can you recover a band-pass signal from its low-pass equivalent? Prove it mathematically.

P2.- Use a phasors diagram to obtain the expressions for the in-phase and quadrature components ($x_i(t)$ and $x_q(t)$); and the envelope and instantaneous phase ($A(t)$ and $\phi(t)$) of the band-pass signal:

$$x(t) = x_1(t)\cdot\cos(w_0 t) + x_2(t)\cdot\cos(w_0 t + \alpha)$$

P3.- Repeat the previous exercise for the signal:

$$x(t) = x_1(t)\cdot\cos((w_0 - w_1)t) + x_2(t)\cdot\cos((w_0 + w_1)t)$$

P4.- A low-pass signal *x(t)* whose Fourier transform is shown in Figure 4(a) passes through the system shown in Figure 4(b) (where *H* blocks represent Hilbert transforms). Assuming that *W* << f_0, represent in frequency the signals obtained at each point of the system.

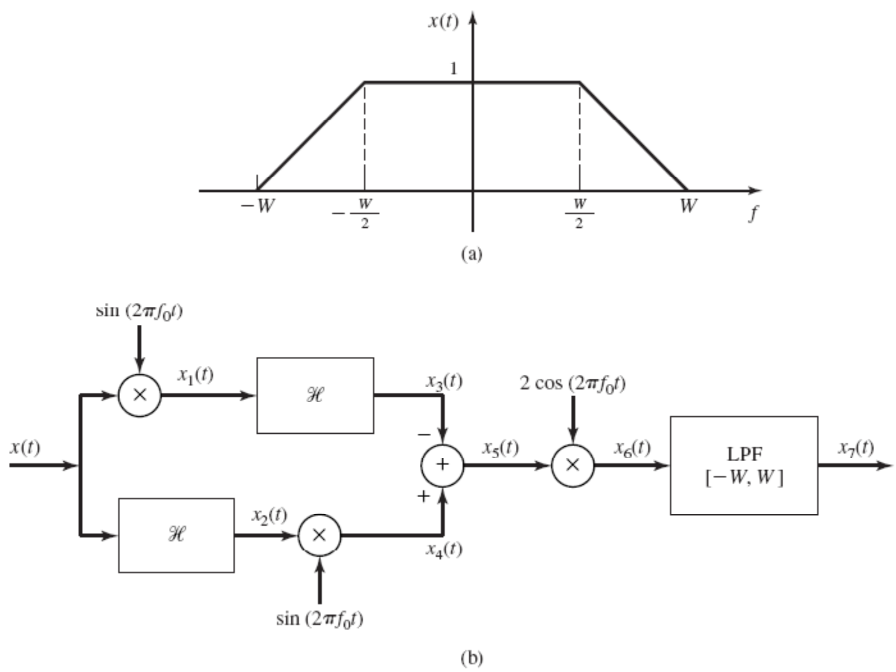

Figure 4.

P5.- Suppose a band-pass signal *x(t)* and a low-pass signal *m(t)* whose spectra do not overlap. Prove that the Hilbert transform of $c(t) = m(t)x(t)$ equals $m(t)\hat{x}(t)$.

P6.- Suppose a low-pass signal $m(t)=\text{sinc}^2(t)$ and a band-pass signal $x(t)=m(t)\cos(2\pi f_0 t)-\hat{m}(t)\sin(2\pi f_0 t)$:

a) Obtain the analytical signal associated to *x(t)* and low-pass equivalent of *x(t)*.

b) Obtain and represent the Fourier transform of the signal *x(t)*. (Remember that $TF\{m(t)\}=\Lambda(f)$.)

c) Repeat the two previous sections with $x(t)=m(t)\cos(2\pi f_0 t)+\hat{m}(t)\sin(2\pi f_0 t)$.

P7.- Assuming a center frequency f_c = 1200Hz, obtain the low-pass equivalent and the in-phase and quadrature components of the signal:

$$X(f)=\begin{cases}1 & 900Hz \le |f| < 1300Hz \\ 0 & resto\end{cases}$$

P8.- Suppose the next signals:

$$g(t), \text{ with } TF\{g(t)\}=\Pi(f)$$

$$x(t)=2\cdot g(t)\cdot\cos\left(2\pi\cdot f_0\cdot t+\frac{\pi}{3}\right) \qquad y(t)=2\cdot g(t)\cdot\sin\left(2\pi\cdot f_0\cdot t+\frac{\pi}{2}\right)$$

$$p(t)=x(t)+y(t)$$

a) Obtain the analytical signal associated to *x*(*t*) and its low-pass equivalent.

b) Obtain the signal $\hat{y}(t)$ (i.e., the Hilbert Transform of *y*(*t*)). Obtain and represent the $TF\{\hat{y}(t)\}$.

c) Use a phasors diagram to obtain the expressions for the in-phase and quadrature components ($p_i(t)$ and $p_q(t)$); and the envelope and instantaneous phase ($A(t)$ and $\phi(t)$) of the signal *p*(*t*).

Note:
$$\sin(A+B)=\sin(A)\cos(B)+\cos(A)\sin(B)$$
$$\cos(A+B)=\cos(A)\cos(B)-\sin(A)\sin(B)$$

P9.- Assuming a center frequency f_c = 1200Hz and the signal:

$$X(f)=\begin{cases}1 & 1100Hz \le |f| < 1200Hz \\ 0.5 & 1200Hz \le |f| < 1350Hz \\ 0 & resto\end{cases}$$

a) Obtain the low-pass equivalent.

b) Obtain the in-phase and quadrature components ($x_i(t)$ and $x_q(t)$).

c) Obtain the signal $\hat{x}_i(t)$ (i.e., the Hilbert transform of the signal $x_i(t)$).

Nota: $\prod\left(f/F\right)\xrightarrow{TF^{-1}} F\cdot\text{sinc}(Ft)$

P10.- A signal $x(t)$ with Fourier transform $X(f) = \prod\left(\frac{f}{1000}\right)$ passes through the system shown in Figure 10 (where H blocks represent Hilbert transforms). Assuming that W = 1KHz and f_0 = 10KHz:

a) Obtain (in time and frequency; only up to the signal $x_5(t)$) and represent in frequency the signals obtained at each point of the system.
b) Obtain the analytical signal associated to $x_5(t)$ and its low-pass equivalent.

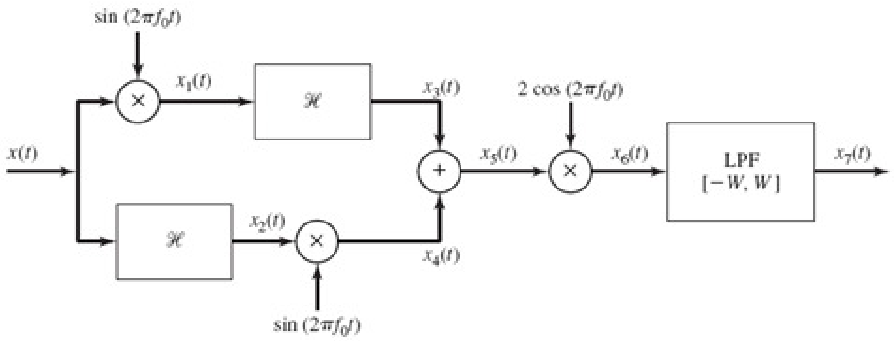

Figure 10.

Note: You can leave the solutions of this problem in terms of $x(t)$ and TH{$x(t)$}.

P11.- Suppose the next signals:

$$g(t),\ \text{with}\ TF\{g(t)\} = \Pi(f)$$

$$x(t) = 2\cdot g(t)\cdot\cos\left(2\pi\cdot f_0\cdot t + \frac{\pi}{3}\right) \qquad y(t) = 2\cdot g(t)\cdot\sin\left(2\pi\cdot f_0\cdot t + \frac{\pi}{2}\right)$$

$$z(t) = -2\cdot g(t)\cdot\cos\left(2\pi\cdot f_0\cdot t - \frac{\pi}{3}\right) \qquad p(t) = x(t) + y(t) + z(t)$$

a) Use a phasors diagram to obtain the expressions for the in-phase and quadrature components ($p_i(t)$ and $p_q(t)$); and the envelope and instantaneous phase ($A(t)$ and $\phi(t)$) of the signal $p(t)$.
b) Obtain the analytical signal associated to $x(t)$ and its low-pass equivalent.
c) Obtain and represent the $TF\{\hat{g}(t)\}$ (i.e., the Fourier transform of the signal $g(t)$).
d) Obtain the signal $\hat{y}(t)$ (i.e., the Hilbert transform of the signal $y(t)$). Obtain and represent the $TF\{\hat{y}(t)\}$.

Note:

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$
$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$